Quantum Ghosts

Quantum entangled particles and Bell's Inequality.
Demonstrating that action at a distance is possible.

Indeterminism; Crown casino wouldn't be in business without it; nor would quantum mechanics. Indeterminism is a familiar enough concept related to probability, but it's whether the universe is inherently probabilistic that is of particular interest. Does God really play dice? Or is it just that we don't have the tools to predict with certainty the outcomes of apparently probabilistic systems?

Matthew Lay.

Quantum mechanics rejects the notion of a deterministic or clockwork universe, setting it aside from classical physics. Indeed the deterministic nature of the universe was the cause of much debate between Einstein and Bohr. In 1935, Einstein together with Boris Podolsky and Nathan Rosen (EPR) published an article questioning the validity of quantum mechanics[1]. In this EPR proposed a thought experiment using Heisenberg's uncertainty principle to show that in a given two-particle system, the measurement of one particle can instantaneously influence the other particle at an arbitrary distance away. Such particles are referred to as quantum entangled particles. Einstein argued that quantum mechanics could not be valid if it allowed the possibility of a non-local interaction, which contradicted special relativity. What was proposed as a criticism however has proven to be one of quantum mechanics' greatest triumphs.

To examine this problem let us first consider the effect of a polariser on photons.

Wave Theory

From the perspective of wave theory when light of polarisation angle ϕ passes through a ±A polariser, the polariser acts by resolving the electric field vector of the light into two orthogonal components (fig. 1). These components emerge in two separate channels (fig. 2).

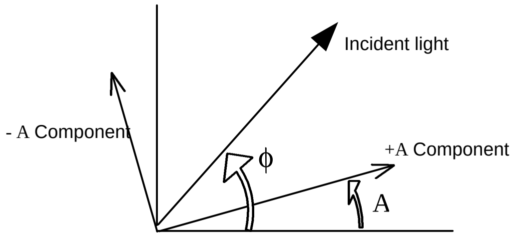


Figure 1 : Components of polarisation. Note that the polarisation of the emergent components depends on the orientation of the polariser (A).

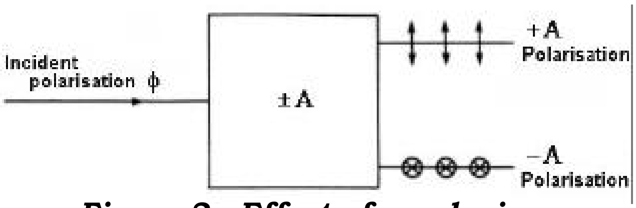


Figure 2 : Effect of a polariser

The intensity of light in the emergent components is given by the square magnitude of the respective electric field components[2]. We find that when the polariser is orientated at 45° both components have an equal intensity.

Photon Theory

What's the story when we lower the incident intensity so that only one photon at a time passes through? Which channel does it come out of? The quantum mechanical interpretation is that the probability of a photon emerging from a channel is proportional to the intensity. So for a 45° polarisation there is an equal probability of emerging from either channel, hence the behaviour of a single photon is completely random and non-deterministic.

Entangled Particles

Entangled particles form a single quantum state which has certain definite properties. In the EPR paradox the system consists of two particles with a total angular momentum of zero. Experimentally however, angular momentum is difficult to deal with. Instead we will consider the polarisations in two photon systems.

Consider a system of two entangled photons (alpha and beta) emitted in different directions from an atom (fig. 3). Alpha and beta may have different frequencies, but what is important is that they are al-

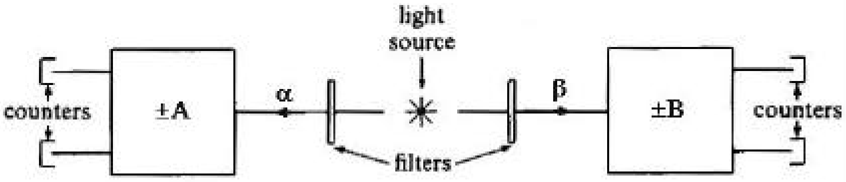


Figure 3 : Entangled two photon polarisation experiment

ways *measured to have* orthogonal polarisations.

The absolute polarisations of alpha and beta are random. But it remains true that when we measure α and then β:

$$\beta = \alpha \pm \pi/2$$

That is, their relative polarisations are fixed.

Non-Locality

Consider using a ±A polariser to measure alpha and beta. Before measuring beta we know that if alpha emerges in say channel +A, then we classically expect beta to emerge in channel -A (the orthogonal channel to + A) with certainty. But quantum theory says there is only a probability for beta to emerge in channel -A. The behaviour of beta is completely random and could emerge from either channel. Yet despite this, *alpha and beta have some sort of correlation such that they are always measured in opposite channels.*

Is it possible that when alpha is measured it signals beta to emerge in the opposite channel, suggesting action at a distance? Or is it that both photons posses a hidden variable telling them to emerge in their given channels, which correspond to orthogonal polarisation? To analyse this we need to use Bell's Inequality.

Bell's Inequality

In 1969 Bell derived an inequality based on local hidden variable theory that assumed the photons behave completely independent of each other. When it comes down to it, Bell's inequality applies to measurements of any three independent variables (which have two possible values) that have random outcomes (eg. Flipping three different coins).

Applying the inequality to our case requires measurement of three different polarisation angles, which isn't possible with a single photon pair. Thus we are forced to consider three separate photon pairs over a large number of pairs (fig. 4). Statistically this gives us the equivalent of having measured three polarisations of one photon pair, since photon pairs have random (absolute) polarisations. Hence over a large number of measurements the differences between pairs will average out.

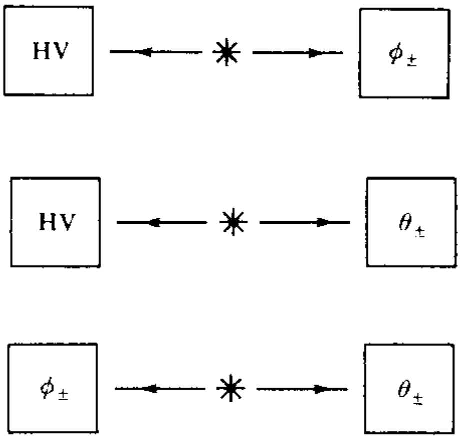


Figure 4 : Polarisation experiments for three pairs of correlated photon pairs.

Bell's inequality states[3]:

$$n(H, +\phi) + n(-\phi, +\theta) \geq n(H, +\theta)$$

Where n(x, y) corresponds to the number of photon pairs detected with alpha emerging from channel x and beta from channel y. HV corresponds to horizontal and verticle orientations.

To determine the predicted n values consider placing a ±A polariser to measure alpha and a ±B polariser to measure beta. ±A polarisers emit photons in channels: +A with polarisation angle A with respect to the horizontal, and channel -A with polarisation orthogonal to +A (cf. fig. 1). Similarly for ±B. Note that the polarisations of +A and +B are parallel to the polariser angles A and B respectively.

Quantum Theory

The probability of a photon emerging in a given channel from quantum theory however, directly relates back to the formulae for the intensities of emergent components in wave theory. Except in place of intensity of light, we talk of the probability of a photon emerging from a given channel.

We consider that the two photons are not only measured to have orthogonal polarisation, but are in fact emitted with it. Now, if alpha is measured in channel +A (ie. with polarisation of A), then beta must have a polarisation (β) equal to A ± $\pi/2$, where $0 \le \beta < \pi$. Now consider +B and -B the components of β (Fig. 5).

We consider N photon pairs passing through the polarisers. Since the absolute polarisation of alpha is random, we expect an even amount to emerge in +A and -A. Hence N/2 alphas have polarisation A. The probability of a photon emerging in a given channel is also proportional to the square magnitude of the electric field vector component.

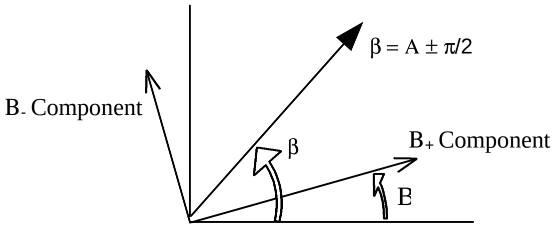


Figure 5 : Components of beta

Using the above concepts, we can calculate the expected number of B_+ and B_- photons and determine our n values:

$$n\,(A_+,B_+) = (N/2)\,\sin^2(B - A)$$
$$n\,(A_+,B_-) = (N/2)\,\cos^2(B - A)$$
$$n\,(A_-,B_+) = (N/2)\,\cos^2(B - A)$$
$$n\,(A_-,B_-) = (N/2)\,\sin^2(B - A)$$

On substitution of our derived values for the various values of A & B into Bell s inequality we have:

$$\cos^2(\phi) + \sin^2(\theta - \phi) \ge \cos^2(\theta)$$
$$\cos^2(\phi) + \sin^2(\theta - \phi) - \cos^2(\theta) \ge 0$$

We note that this inequality is violated for certain polariser angles. For example, the case where $\phi = 3\theta$ is graphed in figure 6:

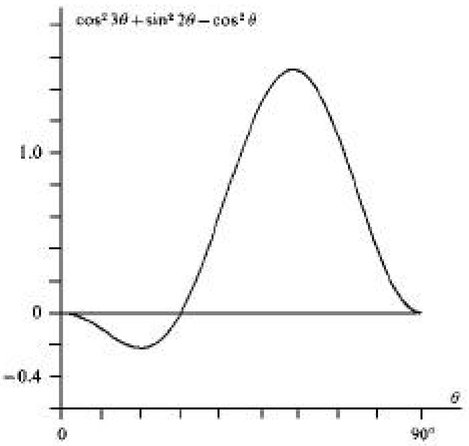


Figure 6 : For** f = 3q, **Quantum theory violates Bell s Inequality for 0** < q < **30°

Since quantum mechanics violates Bell s inequality in at least one way, this implies that the assumption that the photons behave independently of each other is false. They must be in fact linked in some way. Or if you want to be fashionable, they are entangled.

Experimental Confirmation

As you may expect, experiments with photon pairs agree with the results of quantum mechanics. But alas they are not direct tests of the inequality quoted above. There are many practical difficulties that prevent this; one such problem is the fact that polarisers and detectors are never 100% efficient[4].

Clauser, Horn, Shimas and Holt revised Bell s inequality in 1969 to overcome these difficulties, and presented an equivalent form making it easier to test[5]. Many experiments were performed to do so, but it is the 1981 Aspect experiment that is considered to be the most definitive proof that Bell s inequality is in fact violated and that results agree with quantum theory[6]. Furthermore, Aspect s experiment showed that the distance between polarisers had no effect on the results and that if a signal is sent between photons it must travel faster than the speed of light by at least several factors.

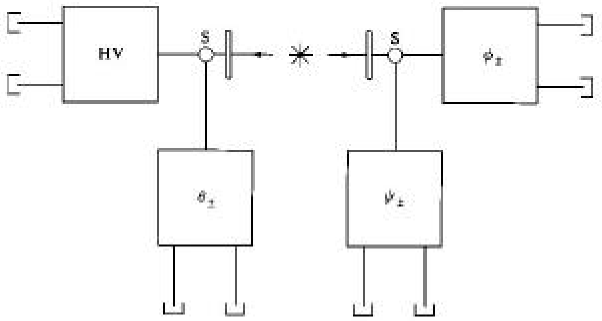


Figure 7 : Schematic of Aspect s experiment.

It is only now however that we have begun to exploit the non-local properties of entangled particles. 1999 has seen the birth of quantum teleportation, the transferal of unique quantum states instantaneously across space. Who knows what the new millennium will produce? More concerning this in future issues.

Please note the web edition of this will be out soon and will contain additional information.

http://www.ph.unimelb.edu.au/PSS/pert.html

Notes & References

[1] The famous EPR paper, Einstein, Podolsky, Rosen, Can the quantum mechanical description of physical reality be considered complete? , Physical Review, vol. 47, p. 777-780, 1935 – reprinted in Wheeler and Zurek, Quantum theory and measurement.

[2] Recall that the polarisation is collinear with the electric field vector.

[3] Rae A., Quantum physics: Illusion or reality? , p. 37, 1986.

[4] Rae A., Quantum physics: Illusion or reality? , p. 42, 1986.

[5] Clauser J. F. et al, Phys. Rev. Lett. Vol. 23, p. 880, 1969

[6] Aspect A. et al, Experimental tests of realistic local theories via Bell s Theorem , Phys. Rev. lett. Vol. 47, No. 7 p. 460, 1981.

The Power of Rational Inquiry and the Human Mind

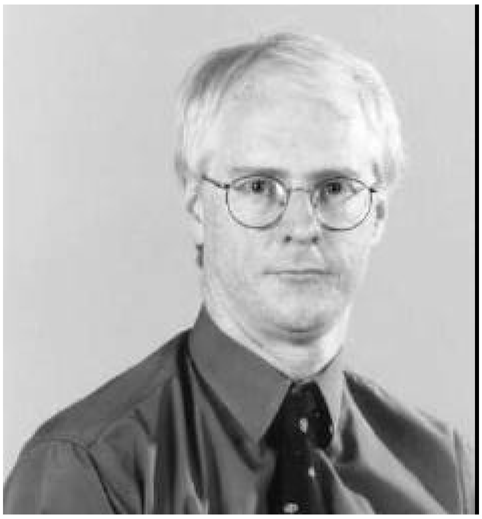
An interview with Professor Keith Nugent.

Up on the seventh floor of the physics building, in an office with carpet on the floor, Professor Keith Nugent, Head of the School of Physics, deals with administrative problems and funding cuts. Yet he still finds time to continue his research, lecture to first and third year students and support the Physics Students Society!

Catherine Bellair.

What is your official position in the School of Physics, and how long have you held it for?

My current position is Head of the School of Physics which I have now held for three and a half years. I am also a Professor of Physics, and have been since 1993. I ve been on the staff of the School since very late 1985.



Why did you decide to become a scientist and a physicist in particular?

I didn t really know what I wanted to do when I was at school. My father was an industrial chemist, so he was always interested in science. I was kind of interested